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Extending Nyquist limit by velocity difference dealiasing rules using a triple PRT scheme

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The staggered trigger technique consists of using non-uniform Pulse Repetition Times (PRT) to mitigate the range-velocity ambiguity. In a triple PRT technique, three different flow velocities are estimated for each PRT. The velocity difference between these three velocities is used to estimate in which Nyquist interval is the real flow velocity. In this work, a triple PRT method is applied to a rotating cylinder flow experiment. By using a PRT scheme 2/3-3/4, where pulse emission occurs as a two, three, and four times a base period, we demonstrate that this method achieves 6 times the maximum conventional velocity. The number of cycles emitted for the triple PRF was studied. For the experimental conditions set, a 6-cycle burst guarantees an accurate velocity estimation. A theoretical relation between Signal to Noise Ratio (SNR) of the signal received and the number of cycles emitted is presented.

Keywords: Staggered trigger, Doppler technique, triple PRF

1. Introduction

The Doppler method consists of estimating a spatial velocity profile of a liquid flow through the echoes of a pulsed ultrasonic transducer [1]. It generally uses a phase-shift or autocorrelation algorithm to extract the velocity from the signal received [2]. This technique has a limitation on the maximum velocity that can be measured. Whenever flow velocity surpasses this limit velocity aliasing occurs [1]. Reducing the period between ultrasound emissions can extend the velocity limit of the technique, however with a proportional reduction in the maximum distance that can be measured. Several dealiasing techniques were proposed to overcome this issue. A multifrequency arrange [3-5], is based on using two ultrasonic transducers of different resonant frequencies to increase the velocity limit. Another approach is based on using a different algorithm for estimating the velocity. The cross-correlation method [6], the extended autocorrelation method [7], and the velocity matched spectrum analysis [8] use different mathematical approaches to estimating velocity that can increase maximum velocity compared to the conventional phase-shift estimator. However, these advanced velocity estimators are computationally more intensive [9,10].

Velocity dealiasing can also be achieved by using a non-uniform period between ultrasound pulses. With this approach, up to 6 times, the conventional velocity could be attained. Thus, it allows measuring large velocities in long-range measurements. Also, since this method uses a long period between ultrasound emissions it will imply in fewer data to process and less complex acquisition hardware. Staggered PRT (Pulse Repetition Time) or dual PRT is characterized by alternating the period between ultrasonic bursts. The first works using this concept were applied for blood flow [11] and after for weather radar [12,13]. The first industrial flow application of the staggered PRT was reported in [13-14]. Velocity estimation was done by an algorithm denominated as a feedback method. They reported that this technique can measure up to 5 times greater than Nyquist limit. A different form of deciding the aliasing factor for the staggered PRT was proposed in [15]. They used the velocity difference from the velocities estimated for each PRT to decide the dealiasing factor. This work was focused on weather radar application and showed that it could reach up to 2 times the maximum conventional velocity regarding the short PRT period. We have adapted the method of [15] and applied it to fluid engineering [16-17]. Simulation results of [16] showed that the technique can reach velocities higher than described in [15] and with low temporal resolution than [14]. However, experimental results of [17] showed that with a 1-cycle ultrasound pulse emission the maximum velocity was limited to 2 times the maximum conventional velocity regarding the short PRT period. More recently, the work of [18] showed that the staggered method can be further extended to a multiple PRT. Their application was focused on medical imaging systems. They reported measuring up to 6 times the Nyquist limit by using a 2/3-3/4 multiple PRT or triple PRT. This was reached using a 6-cycle ultrasound pulse and they also reported a low temporal resolution.

Triple PRT or dual PRT can also be used to extend the maximum distance measured in applications which velocity aliasing is not an issue. A conventional uniform PRT using a phase-shift estimator can be converted to a dual or Triple-PRT by dropping lines of emissions. Increasing the period between emissions implies in a large range of distance measured. For a triple PRT, an increase of 6 times in distance can be attained. Another benefit in this approach, is the reduction of data processing. As an example, a minimum of 6 lines of emissions can be dropped in comparison with the uniform period of emissions. This also implies in using pulse repetition frequency (PRF) roughly 1/6 compared to the uniform period of emissions. Also, reducing the PRF enables the use of a higher voltage pulses for excitation, since transducer dissipation power is proportional to the PRF used.

Inspired by the work of [18] we adapted the triple PRT for industrial application. Using a classical rotating cylinder flow we have assessed the accuracy of this technique. Several numbers of ultrasonics burst cycles were tested to find the appropriate ultrasonic excitation scheme for this technique. We finish, showing a theoretical relation for the technique based on the SNR ratio.

2. Triple PRT method

The base principle of the triple method is to alternate PRT between 3 different periods: T_1, T_2 and T_3, where $T_1/T_2 = m/n$ and $T_1/T_3 = n/p$ and m, n and p are positive integers relatively prime. For this work, $m = 2$, $n = 3$, and $p = 4$, also referred to as 2/3-3/4 scheme. The autocorrelation method is applied three times for each PRT resulting in three velocities estimated v_1, v_2 and v_3 (Figure 1). As in the conventional phase-shift method these three velocities are limited to [1]

$$v_{a1} = \frac{c}{4T_1 f_c}, \tag{1}$$

$$v_{a2} = \frac{c}{4T_2 f_c}, \tag{2}$$

$$v_{a3} = \frac{c}{4T_3 f_c}, \tag{3}$$

also known as Nyquist limit, where c is the sound velocity of the medium and f_c is the transducer central frequency. Each time flow velocity surpasses one of these limits the correspondent estimated velocity will be aliased. The velocity difference $v_{21} = v_2 - v_1$ and $v_{31} = v_3 - v_1$ are used as inputs to a set of dealiasing rules. They are arranged in a lookup table that will result in the aliasing factor n_1, n_2 and n_3 [20]. And the velocities are dealiased by the following relation

$$v_{1d} = v_1 + 2n_1 v_{a1}, \tag{4}$$

$$v_{2d} = v_2 + 2n_2 v_{a2}, \tag{5}$$

$$v_{3d} = v_3 + 2n_3 v_{a3}. \tag{6}$$

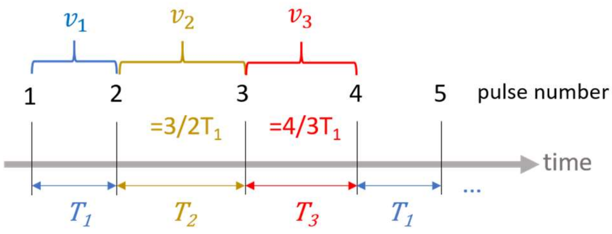

Figure 1: Triple PRT pulsing scheme.

If the technique were a dual PRT, the maximum dealiased velocity measured would be $2v_{a1}$ and $3v_{a1}$, for $T_1/T_2 = 2/3$ and $T_1/T_3 = 3/4$, respectively. However, the combination of the inputs v_{21} and v_{31} is used to increase the maximum dealiased velocity. For $m = 2$, $n = 3$, and $p = 4$ the maximum velocity regarding v_{a1} is least common multiple between m, n, or $6v_{a1}$ [18].

2.1 Minimum SNR

According to [18] the maximum allowable error of velocity estimates for a 2/3-3/4 triple scheme is given by

$$|e_{\max}| = \frac{v_{a1}}{\sqrt{12} + 4}. \tag{7}$$

In [21] it was derived a relationship between SNR and frequency standard deviation for the autocorrelation algorithm as

$$SNR = \frac{1}{\sqrt{2x^2 e^{-x^2}\left(\frac{N\sigma^2}{F_\sigma^2}\right) + e^{-x^2} - 1}} \tag{8}$$

Where it was assumed that the spectrum of the signal is Gaussian in shape, F_σ is the frequency spread, N denote the number of lines used for estimation, and $x = 2\pi F_\sigma T$ and T is the period between emissions. This equation was derived for the conventional autocorrelation method. However, triple PRT estimation can be treated as three different autocorrelation estimators, each one with its own period between emissions.

Let us assume that this standard deviation, σ, will result in the error in the triple PRT, thus $\sigma \leq |e_{\max}|$, and Eq. (8 becomes

$$SNR = \frac{1}{\sqrt{2x^2 e^{-x^2}\left(\frac{4f_c^2 N {e_{\max}}^2}{c^2 F_\sigma^2}\right) + e^{-x^2} - 1}}, \tag{9}$$

where $(2f_c/c)^2$ was added to translate the maximum velocity error in a maximum Doppler frequency error, f_c is the transducer central frequency and c is the sound velocity.

3. Measurement Method

A cylinder, submerged in a water tank, was filled with 1.8

L of a density match solution (water and glycerol). A 0.5 g of trace powder (EMS GRILTECH 1A P82) with 1.07 g/cm^3 was added to the solution. A 4 MHz ultrasonic transducer (Met-flow) was positioned at 31.5 mm from the central axis (Figure 2). This set-up is a classical rotating cylinder flow with a uniform spatial velocity profile [1].

Cylinder reference velocity is measured by an encoder. Ultrasound emission and acquisition are sampled at 40 MHz by proprietary hardware. Data acquired is processed offline by a personal computer.

Transducer emissions of 2,4,6 and 8-cycles bursts were used in the experiment. For each condition, 18 different angular velocities, ranging from 0.73 to 9.3 rad/s (7 to 89 RPM) were tested. A total of 2 seconds of data was acquired for each velocity. The PRT was set to 2,25 ms (T_1), 3.375 ms (T_2) and 3 ms (T_3) resulting in a PRT ratio of $T_1/T_2 = 2/3$ and $T_1/T_3 = 3/4$. In this condition, the conventional maximum velocities are $v_{a1} = 47.5\ mm/s$ or 14.4 RPM, $v_{a2} = 31.7\ mm/s$ or 9.6 RPM and $v_{a3} = 35.6\ mm/s$ or 10.8 RPM.

Due to the nonuniform pulse repetition period used, the data were filtered to remove stationary echoes using a second-order polynomial regression filter (length M_f=20) [20]. Velocities data were calculated for every 51 emissions resulting in a temporal resolution of 0.144 s. Since velocity is estimated for each PRT, the three velocities are estimated using only 17 pulse emissions each. The spatiotemporal velocity maps were post-processed using a median filter with 3x3 matrix size. Range gate length was set as $4N_{cycle}\lambda$, where N_{cycle} is the number of cycles burst and λ is the wavelength.

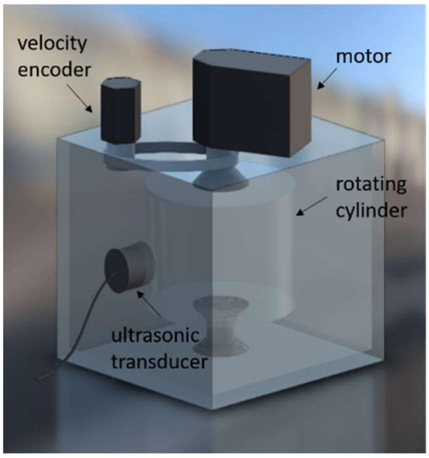

Figure 2: Rotating cylinder flow experimental set-up.

4. Results

The triple PRT overall performance was evaluated by the mean value of the spatial velocity profile (Figure 3). This result was computed for cylinder velocities varying from $0.49v_{a1}$ up to from $6.2v_{a1}$ (approximately 7 RPM up to 89 RPM). The PRT ratio of 2/3/4 has a theoretical limit of $6\,v_{a1}$. Measurements were done using a 6-cycle burst emission. It can be noticed that the technique can estimate the cylinder velocity with less than 5% of error up to the theoretical velocity limit ($6v_{a1}$).

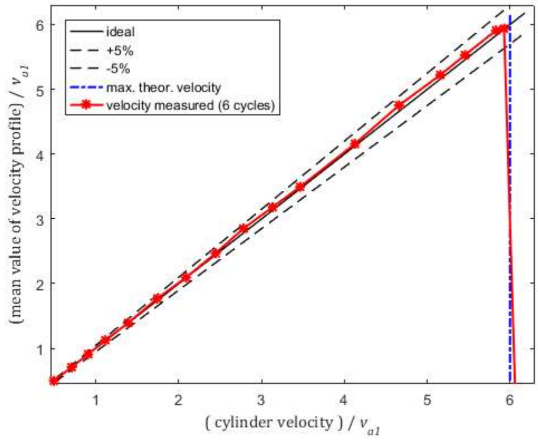

Figure 3: Normalized spatial mean velocity measured versus normalized cylinder reference velocity for a 6-cycle ultrasound emission.

The accumulated root mean square error for the spatiotemporal velocity map was computed for 2,4,6 and 8-cycles. This analysis aimed to evaluate the accuracy of each burst emission (Figure 4). The 2-cycles burst presented a poor result, therefore is not recommended for the 2/3-3/4 technique. The 4,6 and 8-cycles emission presented a similar result, but for velocities in the vicinity of $6v_{a1}$ the 4-cycles emission results in a high RMSE value of 145 mm/s meanwhile the 6-cycles and 8 cycles bursts result in RMSE of 31 mm/s and 38 mm/s, respectively.

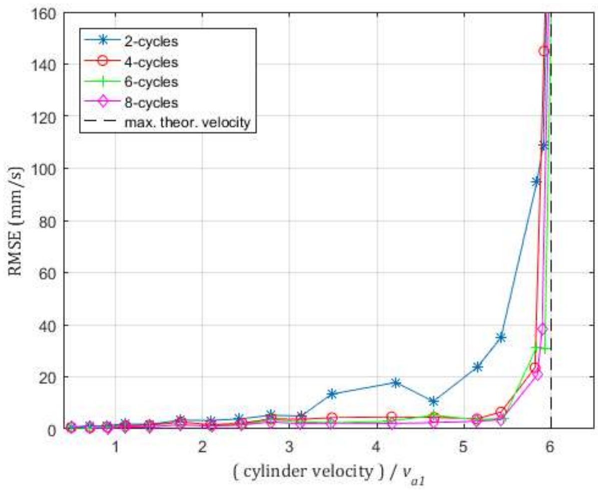

Figure 4: Global root-mean-square error as function of normalized cylinder velocity for 2,4,6 and 8-cycle ultrasound emission.

4.1 SNR Results

Eq. (9) was applied for the experimental setup proposed. It was considered the worst-case scenario, which was $T = T_1 = 2.25\ ms$. Frequency spread was obtained by [1]

$$F_\sigma = \frac{2 f_c v_{a1}}{N_{cycles}} c, \qquad (3)$$

where f_c is the transducer central frequency and N_{cycles} is the number of ultrasound pulses. The number of lines used was $N = 17$. Figure 5 show the results of the dependency of the SNR as the number of cycles are increased. From 2-cycles to 6-cycles burst the SNR decreases 11.7 dB. As the number of cycles increases the spectrum´s frequency spread decreases (Eq. (3)). Narrowing the Doppler spectrum implies in more accurate frequency estimate (or velocity estimate) and a smaller frequency standard deviation value. It explains why the number of cycles influence in the technique accuracy. Fig. 5 also serves as a specification for experimental setup. It should be observed that it is neglected that the increase in the N_{cycles} could also increase the signal power thus increasing SNR.

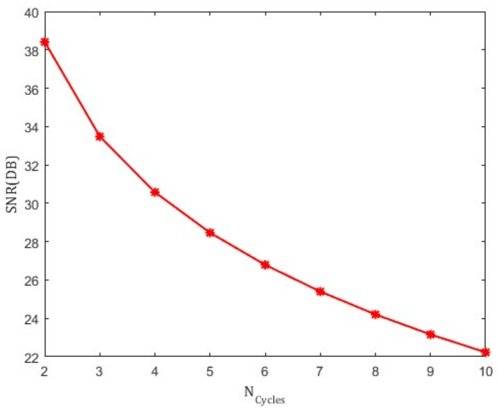

Figure 5: SNR relationship with the number of emitted cycles.

5. Summary

The triple PRT technique 2/3-3/4 implemented showed that it can measure velocities up to 6 times the Nyquist limit. To reach this same result with a dual PRT, it would be necessary to use a PRT ratio of $T_1/T_2 = 6/7$. At this ratio, the relative mean square error increases significantly [18].

The number of ultrasonics cycles emitted was studied. For the experiment described, a 6-cycles ultrasound emission should be used to guarantee accurate measurement in the vicinity of Nyquist limit. It should be noted that the increase of the ultrasound burst will directly increase the spatial resolution of the velocity profile.

A theoretical equation for the SNR was presented. It shows the relation of the SNR with the number of cycles emitted. This relationship could be studied further using simulated ultrasound signals where is possible to have accurate SNR values. In typical applications of Doppler technique, the margin for increases SNR ratio is the use of a higher pulse voltage excitation or increasing the number of cycles emitted.

6. Future work

We plan to conduct additional studies using simulation of ultrasound signals to fully understand the relationship between SNR and the number of pulses emitted.

The triple PRT code will be adapted to Python language and will be available for free as a Python library at https://github.com/cesarofuchi/pyuvp.

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